V. "On the Direct Application of First Principles in the Theory of Partial Differential Equations." By J. LARMOR, M.A., Fellow of St. John's College, Cambridge. Communicated by Lord RAYLEIGH, D.C.L., Sec. R.S. Received November 8, 1887.

(Abstract.)

If an equation involving total differentials of any number of variables can be expressed in the form—

$$\delta u + \sigma\, \delta v = 0,$$

where u, v are any functions of the variables, then the only *single* integral algebraic relations that are consistent with it are included under the form—

$$u = \phi(v).$$

When the form of σ is assigned, the functional symbol ϕ is to be chosen, if possible, so as to agree with that form; and if this is not possible, then the equation has no integral expressible as a *single* relation. This statement holds because the equation expresses a particular case of the proposition that if $\delta u = 0$, then $\delta v = 0$, and conversely, *i.e.*, that u remains constant (does not vary) when v is constant, and only then, whatever be the particular values assigned to the variables: but this is simply the definition of the algebraic idea of functionality.

If, however, σ involve differentials, the alternative $\delta u = 0$ when $\sigma = 0$ may lead to integrals of a new type.

In the same way, an equation of the form—

$$\delta u + \sigma_1\, \delta v + \sigma_2\, \delta w = 0,$$

must have all its *single* integrals included under the form—

$$u = \phi(v, w),$$

where the form of ϕ is to be chosen so as to agree with the expressions for σ_1, σ_2, when these are assigned.

When no *single* integral exists, equations of this type may be satisfied by two simultaneous integral relations, one of which may be arbitrarily assumed, as originally pointed out by Monge. This kind of exception, however, need not trouble us when partial differential coefficients are concerned; for these implicitly assume the existence of a single relation connecting the dependent variable with the independent ones.

Traces of this idea are to be found throughout the writings of Boole

—and of Monge long previously. In this paper it is applied, first to the non-analytical exposition of the differential criteria of algebraic functionality given by Jacobi, and then to the discussion in a similar manner of the theory of partial differential equations of the first and second order, particularly those named after Lagrange, Monge, and Ampère.

VI. "On the Power of Contractility exhibited by the Protoplasm of certain Plant Cells." (Preliminary Communication.) By WALTER GARDINER, M.A., Fellow of Clare College, Cambridge, Demonstrator of Botany in the University. Communicated by Prof. M. FOSTER, Sec. R.S. Received November 21, 1887.

In a former communication ('Roy. Soc. Proc.,' No. 240, 1886), some account was given of the principal changes which take place in the gland cells and stalk cells of *Drosera dichotoma* during secretion. The present paper deals with certain experiments and observations which were undertaken in order to attempt to ascertain by what mechanism the bending of the tentacles is made possible in *Drosera*, and what changes occur in the tentacle cells.

During actual movement no obvious histological changes can be detected in the cells of the bending portion, but when the tentacle has become well inflected, it becomes apparent that the cells of the convex side become more, and those of the concave less turgid than before. Some time after stimulation, and when the period of aggregation has set in, it can be observed that the cells of the convex side are less aggregated than those of the concave. Having ascertained that of the dye solutions, eosin, and of salts, the salts of ammonia, are readily sucked up into the tissue, it was further noticed that in stimulated tentacles the cells of the convex side readily allow the solutions to penetrate, while those of the concave are only penetrated with great difficulty. Thus in the case of a stimulated tentacle treated with eosin, the convex cells are stained long before the concave, and with ammonic carbonate the tannin of the convex cells may be precipitated while the concave cells remain normal, or the convex cells may even be killed while the concave cells remain alive. Thus after stimulation certain changes have occurred in the concave cells of the bending portion, and one result of this change is an increased impenetrability of the primordial utricle. In my former paper I have shown that the tentacle cells of *Drosera* are very sensitive to contact, for if the gland cells be slightly crushed, all movement of the stalk cells ceases for a time, and the spindle-shaped rhabdoid contracts and tends to become spherical. Bearing in mind also the very pro-